Math 555, Fall 2011 J. Rauch
Homework 7 Due November 1, 2011

The first four problems of this assignment are aimed at addressing weaknesses revealed in the midterm exam.

1. 150/8. First four terms is sufficient.

2. 168/8.

3. Consider the mapping $f(z) = z^2$. Denote by θ the polar angle with $-\pi < \theta < \pi$. Define two open domains,

$$\Omega_1 \ := \ \Big\{ 1 < |z| < 2, \ \ \frac{-\pi}{2} < \theta < \frac{\pi}{6} \Big\}, \qquad \Omega_2 \ := \ \Big\{ 1 < |z| < 2, \ \ \frac{-\pi}{2} < \theta < \frac{4\pi}{6} \Big\}.$$

a. Sketch the two domains.

b. Show using the Inverse Function Theorem that f is locally invertible with analytic inverse on a neighborhood of each point $z \neq 0$.

c. Show that f is a one to one invertible map of Ω_1 onto its range, and that the inverse is analytic. Be sure to sketch the range.

d. Show that f is **not** a one to one map of Ω_2 onto its range.

Discussion. These examples illustrate an important fact about learning mathematics. In order to understand theorems, one needs counterexamples that show why the hyptheses are needed and/or the conclusions are limited.

Such examples are part of the universal currency of mathematics, *e.g.* smooth functions with a point where all derivatives vanish, and, pointwise but not uniformly convergent sequences of functions, ... *etc.* Locally but not globally invertible maps belong on the list.

5. 167/4e.

6. Suppose that $f(z)$ is analytic in $|z| < R$ and $0 < r < R$. Use the residue theorem to evaluate the integral

$$\int_{|z|=r} \frac{f(z)}{(z-a)(z-b)} \ dz$$

where z and b are distinct points with modulus less than r. **Discussion.** This is the integral that is supposed to appear on the right in 152/15. I am assigning it as an application of the residue theorem instead.

7. 167/4c.