

1. Compute using residues

$$\int_0^{\infty} \frac{1}{x^4 + 1} dx.$$

Ans. $\pi/(2\sqrt{2})$.

2. Find the singular parts of the Laurent expansions of

$$f(z) = \frac{z}{1 + z^3},$$

at its poles. Use those results to deduce the partial fraction expansion. **Hint.** Handout on partial fractions.

3. Use the result of the preceding problem to compute using antiderivatives,

$$\int_0^{\infty} \frac{x}{1 + x^3} dx.$$

Discussion. This is an example where the antiderivative works and the standard residue method does not. **Warning.** Be careful about branches of logarithms. You need them to be defined on the positive real axis.

4. Compute

$$\int_0^{\infty} \frac{x \sin 2x}{x^2 + 3} dx.$$

Ans. $(\pi/2)e^{-2\sqrt{3}}$.

5. Evaluate using residues

$$\text{P.V.} \int_{-\infty}^{\infty} \frac{x^3 + 1}{x^4 + 1} dx.$$

Hint. For large z , write $z^3 + 1 = z^3(1 + 1/z^3)$ and similarly the denominator. Find that

$$\left| \frac{z^3 + 1}{z^4 + 1} - \frac{1}{z} \right| \leq \frac{C}{|z|^2}. \quad (1)$$

Discussion. This hint would not be necessary had we worked an example like this in class. At infinite (as in this case) or finite values of x where principal values are required, this sort of manipulation allows one to replace integrands by their leading approximation. For this problem the error in (1) is smaller than advertised. It would be $\sim 1/|z|^2$ if there were a z^2 term in the numerator or a z^3 term in the denominator.

6. $187/4$.

7. For $\omega < 0$ compute

$$\int_{-\infty}^{\infty} \frac{\cos \omega x}{x^2 + 1} dx.$$