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- Use the method of Homework 8, Problems 1,2 to evaluate

$$\sum_1^{\infty} \frac{1}{1+n^2}.$$

Hints. Using a rectangle instead of a circle is a little easier.

- Evaluate

$$\int_0^{\infty} \frac{z^{1/3} \log z}{z^2 + 1} dz,$$

where both $z^{1/3}$ and the logarithm are positive and real on $]0, \infty[$. **Ans.** $\pi^2/6$.

4. i. Find the image by the mapping $w = \sin z$ of the vertical half rays $\{x + iy : y \geq 0\}$ with $x = -\pi/2$, $x = 0$, and $x = \pi/2$.

ii. Show that $\sin z$ is a one to one function on the strip $-\pi/2 < x < \pi/2$, $0 \leq y \leq \infty$.
Hint. Do **i** and **ii** with bare hands using e^{iz} .

iii. Use these two results to identify the image of the conformal map $w = \sin z$ of the strip from **ii**.

5. Find a conformal map from the sector $0 < \arg z < \pi/3$ to a disk of radius 1. The argument takes values in $] -\pi, \pi[$.

6. Find the image of the strip $0 < y < 1/(2c)$ under the transformation $w = 1/z$. Sketch the correspondence of the boundaries including orientations.

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