Math 555, Fall 2013 **J. Rauch**
Homework 13 **Due Tuesday December 10**

1. This problem is a continuation of Problem 3 on Assignment 12. Find all flows in the wedge whose flow is parallel to the bounding lines and satisfies the symmetry (1) from the preceding assignment. **Hint.** Use the result from that problem. **Discussion.** The flow velocity is bounded at the corner but the derivatives of the velocity are unbouded at the corner. The case of $A = \pi/4$ does not have this divergence.

2-3. i. Starting with flow in the unit disk swirling about the origin, find a swirling flow in the upper half disk so that the flow swirls about the point midway between the circle center and the circumference. **ii.** Show that near the corners of the half disk the flow resembles the flow with complex potential z^2 in a quadrant. **Hints. i.** Map the domains. Then arrange that the point $0 + i/2$ goes to the center by performing an additional self map of the disk. The self maps of the disk were given in class. **ii.** Find the leading term in the Taylor expansion of the complex potential at the corner. The potential of the transformed problem is the transform of the original potential.

4. i. For integer $n \geq 1$ show that the irrotational, incompressible, planar fluid flow with complex potential $F(z) = z^n$ is tangent to the boundary of the wedge $0 < \arg z < 2\pi/n$ and each of its rotates by $k2\pi/n$ with $k \in \mathbb{Z}$.

ii. Sketch the streamlines. **Hint.** The streamlines satisfy $\operatorname{Im} z^n = c$. So z belongs to the image of a $\{\operatorname{Im} w = c\}$ by the appropriate branch of $z = w^{1/n}$.

iii. When n is even, show that the flow is tangent to the boundary of $\{y > 0\}$.

5. 243/3.

6. 243/4.

7. Exercise 2.2 of Fluid Flows handout.