Math 555 Fall 2013 Prof. J. Rauch
Homework 4 Due October 3

1. 74/17.

2. 75/21.

3. 75/25.

4. 75/26.

5. If $f(z)$ is analytic on a neighborhood of $z = 0$ show that

$$\frac{f^{(n)}(0)}{n!}\, z^n \;=\; \sum_{j+k=n} \frac{\partial^j \partial^k f(0)}{\partial x^j \partial y^k}\, \frac{x^j\, y^k}{j!k!}\,.$$

Discussion. This shows that the Taylor series of f at the origin in the sense of complex analysis is identical to the Taylor series as an infinitely differentiable function of x, y.

6. 74/18. Take $a = 0$.

7. Let S denote the square with vertices at ± 1 and $\pm i$. For each of the following analytic functions, find the points of S where $|f(z)|$ attains its maximum values.

$$\textbf{a}.\ f(z) \;=\; \sin z\,, \qquad \textbf{b}.\ f(z) \;=\; e^z\,, \qquad \textbf{c}.\ f(z) \;=\; \frac{1}{z-(1+i)}\,.$$

Discussion. These are all examples illustrating the maximum modulus principal.

8. i. Suppose that $f : \{0 < |z| < 1\} \to \mathbb{C}$ is analytic on the punctured unit disk. Suppose that C_1 is a circle in the punctured disk turning once about the origin in the positive sense. Suppose that C_2 is the boundary of a rectangle R with origin in its interior and its boundary in the puctured disk. Show that $\oint_C f(z)\, dz$ around each of these curves yields the same number. **Hint.** Cauchy's Theorem used more than once on domains that are not simply connected.