Math 555 Fall 2013 Professor J. Rauch
Homework 5 **Due October 10**

1. Derive the Taylor expansion of $f(z) = 1/(1-z)$ about the point $\underline{z} = i$. **Hint.** Write $1 - z = (1-i) - (z-i)$. Factor the larger summand $1 - i$.

2. 91/5. Give proof or counterexample.

3. 152/16.

4. The Taylor expansion

$$\frac{1}{w} \;=\; \sum_{n=0}^{\infty} (-1)^n\,(w-1)^n\,, \qquad |w-1|<1\,,$$

is derived by substitution as in Problem 1. Since the disk $\{|z-1|<1\}$ is simply connected the function has a unique antiderivative F defined on in $|w-1|<1$ and satisfying $F(1)=0$.
i. Integrate the uniformly convergent series over any contour in $|w-1|<1$ connecting $w=1$ to w find the Taylor series of F.
ii. From $F'(w) = 1/w$ with $F(1)=0$ identify the function F.

5. 136/10a, b. **Remark.** Your explanation is important.

6. 213/17.

When computing Taylor series the formula $f^{(n)}(\underline{z})/n!$ for the coefficients is often not an efficient way to compute. It always pays to try to reduce calculatiions to known series like the exponential, trigonometric functions, and the geometric series. For example one should compute Taylor series of $e^z \sin z$ by multiplication. The series of e^z at $\underline{z}$ by writing $e^z = e^{\underline{z}} e^{z-\underline{z}}$. The next problem shows another good technique.

7. Compute the first terms of the Taylor series of $\tan z$ about $\underline{z}=0$ as follows. Write the answer with undetermined coefficients a_j,

$$\tan z \;=\; \frac{\sin z}{\cos z} \;=\; a_0 + a_1 z + a_2 z^2 + \cdots\,.$$

Multiply through by $\cos z$. Expand sin and cos. Deterimine a_j for $j \le 5$ starting with a_0 so that the coefficients of the powers of z on the two sides coincide. **Discussion.** This method of undetermined coefficients is almost always faster than computing derivatives, *i.e.* $d^n \tan z/dz^n$.