Math 555 Fall 2013 — Prof. J. Rauch

Homework 6 — **Due Tuesday October 22**

There is a midterm exam on Thursday 17 October and this homework is NOT due then. We switch to Tuesday due dates to avoid a simultaneous exam and homework. It also solves the problem of Thanksgiving that falls on Thursday. My office hours on Wednesday are now way before the homework due dates. On weeks when it is possible, I will move office hours to Monday 2-4. Try to ask me on Thursdays to tell you in advance.

**Hint.** Use stealth and cunning to compute series. Resort to general formulas for the coefficients is a method of last resort. Always try to derive new series from the basic ones for example $1/(1-z)$ and $e^z$ by differentiation, integration, substitution, division, ... *etc.*

**1.** Insert $z = re^{i\theta}$ in $(1-z)^{-1} = \sum z^n$ to derive the Fourier series

$$\sum_{n\geq 1} r^n \cos n\theta \;=\; \frac{r\cos\theta - r^2}{1 - 2r\cos\theta + r^2}\,.$$

**2.** 104/2.

**3.** Find the Laurent expansion of $1/(z-1)(z-2)$ valid in $|z| > 2$.

**4. i.** The function

$$f(z) \;=\; \frac{1}{z^2(1-z)}$$

has four Laurent expansions. Sketch the regions where each of the four expansions are valid.

**ii.** Compute the partial fraction decomposition of $f$ by computing two of Laurent expansions. Specify the regions in which the expansions are valid.

**5.** 137/21. Your explanation is important.

**6.** Use long division to show that the Laurent series of $1/\sin z$ valid in $0 < |z| < 1$ begins

$$\frac{1}{\sin z} \;=\; \frac{1}{z} \;+\; \frac{z}{3!} \;+\; \Big[\frac{1}{(3!)^2} - \frac{1}{5!}\Big] z^3 \;+\; \cdots\,.$$

**7.** 152/21.