Math 555, Fall 2013 | J. Rauch
Homework 7 | Due October 29, 2013

1. 150/8. First four terms is sufficient.

2. 168/8.

3. Consider the mapping $f(z) = z^2$. Denote by θ the polar angle with $-\pi < \theta < \pi$. Define two open domains,

$$\Omega_1 \ := \ \Big\{ 1 < |z| < 2, \ \ \frac{-\pi}{2} < \theta < \frac{\pi}{6} \Big\}, \qquad \Omega_2 \ := \ \Big\{ 1 < |z| < 2, \ \ \frac{-\pi}{2} < \theta < \frac{4\pi}{6} \Big\}.$$

a. Sketch the two domains.

b. Show using the Inverse Function Theorem that f is locally invertible with analytic inverse on a neighborhood of each point $z \neq 0$.

c. Show that f is a one to one invertible map of Ω_1 onto its range, and that the inverse is analytic. Be sure to sketch the range.

d. Show that f is **not** a one to one map of Ω_2 onto its range.

Discussion. These examples illustrate an important fact about learning mathematics. In order to understand theorems, one needs counterexamples that show why the hyptheses are needed and/or the conclusions are limited.

Such examples are part of the universal currency of mathematics, *e.g.* smooth functions with a point where all derivatives vanish, and, pointwise but not uniformly convergent sequences of functions, . . . *etc.* Locally but not globally invertible maps belong on the list.

4. 167/4c.

5. 167/4e.

6. Find the singular parts of the Laurent expansions of

$$f(z) \ = \ \frac{z}{1 + z^3}\,,$$

at its poles. Use those results to deduce the partial fraction expansion. **Hint.** Handout on partial fractions. Save your work. This will be used in the next assignment.

7. Compute using residues

$$\int_0^\infty \frac{1}{x^4 + 1}\, dx\,.$$

Ans. $\pi/(2\sqrt{2})$.