Math 555, Fall 2013 **J. Rauch**
Homework 9 **Due November 12**

1-2. 187/4.

3. Prove the following result.

Theorem. *If $p(z)$ is a polynomial of degree n,*

$$p(z) \;=\; a_n z^n \;+\; \cdots \;+\; a_1 z + a_0, \qquad a_n \neq 0,$$

then there is an $R > 0$ so that for all w with $|w| > R$ the equation

$$p(z) \;=\; w$$

has n **distinct** *roots.*

Hints. i. Use Rouché's Theorem with $f = a_n z^n$. **ii.** You must choose your own contour(s). **iii.** Show that for large w, $f(z) = w$ has n simple roots far from each other. Show that $p(z)$ has nearby simple roots by choosing appropriate disks centered at the roots of f and showing that p has one root in each. **iv.** If you really want to optimize, try to see how close the solutions are to the solutions of $f(z) = w$. It is not hard to show that the distance between the roots of f and p is $\le C$ for all $|w| \ge R$. You do not need to prove this sharper result.

4. If $c \in \mathbb{C}$ with $|c| > e$ show that the equation $c\, z^n = e^z$ has n roots counting multiplicity inside the unit circle.

5. Let Ω_R denote the part of the disk of radius R and center 0 belonging to the angular sector $0 < \theta < 2\pi/3$. Using the contour $\partial\Omega_R$ evaluate the integral

$$\int_0^\infty \frac{1}{x^3+1}\, dx \;=\; \frac{2\pi}{3\sqrt{3}}\,.$$

Discussion. This shows that with cunning you can do some integrals that look like the residue method does not work. Also the example done with antiderivatives on the last assignment.

6. 190/23a.

7. 190/23d.