Math 558 Fall 2014 Prof. J. Rauch
Homework 10 Due Tuesday November 18

1. Sketch the level surfaces of $E(x, x')$ for Problem 6 on the mechanical systems problem handout. At what energies does the qualitative behavior of the orbits change. Indicate stable and unstable manifolds where that appear. Explain your reasoning. **Hint.** The undamped nonlinear pendulum serves as a model.

2. Answer the same questions for Problem 5 of the same handout.

3. Exercise 3.2 from the Conjugacy handout.

4. 72/5. In this problem conjugacy means linear conjugacy.

We will not cover null clines in class. The material is easy. It is well covered in the section 9.4 of Hirsh Smale and Devaney. And in many elementary courses. The next problems should teach you what you need to know.

5-6. Consider $X' = AX$ with

$$\textbf{a}.\ A = \begin{pmatrix} -2 & 0 \\ 0 & +3 \end{pmatrix}, \qquad \textbf{b}.\ A = \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix}, \qquad \textbf{c}.\ A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Sketch the nullclines and the compass rose directions in each subregion for each of these problems. By compass rose I mean in each subregion draw an arrow pointing northeast, southeast, northwest, or southwest indicating the direction of the vector field. The left hand example in Figure 9.4 of Hirsch-Smale-Devaney gives the idea. Show how this information matches well the qualitative behavior of the orbits.

7. 210/1a.