Math 558 Fall 2014 Prof. J. Rauch
Homework 13 **Never due**

1. Define the trough map $g : [0,1] \mapsto [0,1]$ by

$$g(x) \ := \ -2x+1, \quad 0 \le x \le 1/2, \qquad g(x) \ := \ 2x-1, \quad 1/2 \le x \le 1.$$

Sketch the map. Construct an explicit conjugacy between the trough map and the tent map.

2. The map from $[0,1]$ to itself whose graph is a tent with vertex off center is conjugate to the map whose graph is the tent with vertex at the center. Show that the conjugacy must map $0 \mapsto 0$. Show that the conjugacy cannot be a linear map on any neighborhood of 0. **Hint.** Compute $h^{-1} \circ f \circ h$ near 0.

3. 354/1.

4. 354/2. Where possible, discuss the stability of the elements before and after bifurcation.

5. 354/3. **Discussion.** This is an easy one.

6. 355/6.

7. 355/7. **Discussion.** This is the instability part of the derivative test for stability of a fixed point.

8. 355/8.

9. a. Suppose that f and g are maps of the closed bounded interval I to itself that are conjugate by h. Show that for any $\underline{x}$, the orbit of f through $\underline{x}$ is mapped to the orbit of g through $h(\underline{x})$.

b. Show that if f and g are strictly increasing homeomorphisms, then the values of a conjugacy $h(x)$ for $x \in I \setminus [\underline{x}, f(\underline{x})]$ are determined by the values of h for $x \in [\underline{x}, f(\underline{x})]$.

c. Suppose that $\underline{x}$ and $\tilde{\underline{x}}$ are interior points of I . Suppose that k is an increasing homeomorphism mapping $[\underline{x}, f(\underline{x})]$ to $[\tilde{\underline{x}}, g(\tilde{\underline{x}})]$. Prove that there is a unique conjugacy h whose restriction to $[\underline{x}, f(\underline{x})]$ is equal to k.

Discussion. Note the contrast to the logistic map whose dense set of cycles left no leeway in the definition of a conjugacy to the tent map.