Math 558 Fall 2014 Prof. J. Rauch
Homework 2
Due Tuesday September 16

1. 18/10.

Remark. Section 10 of the Dimension 1.5 handout was written to help with the next question. You may use the Theorems from that section.

2. 18/15. **Hint.** Note that $p < q$ is given. Denote $p(x) := \phi(T, x)$ the Poincaré map. Find the sign of $p(x) - x$ for $x = p, q$. Toward that end show that the solution with $x(0) = p$ must satisfy $x(t) > p$ for $t > 0$. For $0 < t << 1$ prove and use $x'(0) > 0$. If it is violated there would be a smallest $\underline{t} > 0$ with $x(\underline{t}) = p$. Derive a contradition. **Hint** Draw a picture. **Discussion.** The conclusion is valid if the hypotheses are weakened to $f(t, p) \geq 0$ and $f(t, q) \leq 0$ provided that the periodic orbit is required only to satisfy $q \leq x \leq p$. You need not prove that harder result. It is useful in problem 18/16 of HSD that will be assigned later.

3. Exercise 2.2 of the Dimension 1 handout.

The next two problems concern the logistic equation $x' = ax(1 - x)$ with $a > 0$.

4. a. Find the linearized equation[1] at the equilibrium $x = 0$. Explain why this suggests instability of the equilibrium.

b. Find the linearized equation at the equilibrium $x = 1$. Explain why this suggests stability of the equilibrium.

5. The solution of the initial value problem (see text page 5)

$$x' = ax(1 - x), \quad x(0) = A \qquad \text{is} \qquad x(t) = \frac{A\,e^{at}}{1 - A + A\,e^{at}}.$$

c. Find the linearized equation at this solution.

6. Exercise 8.1 of the Dimension 1.5 handout. **Remark.** The online handout "The Steps of Perturbation Theory" may be useful.

7. 17/5.

[1] The **linearization** or **variational equation** of $X' = F(t, X)$ at a solution $\underline{X}(t)$ is the homogeneous linear system $U' = A(t)U$ with $A(t) = D_X F(t, \underline{X}(t))$.