Math 558 Fall 2014 Prof. J. Rauch
Homework 3 Due Thursday September 25, 2014

1. 18/16. **Hint.** Use the discussion before and after Problem 2 of Homework 2.

2. Free fall with small nonlinear friction. A particle of mass m falls freely under the force of gravity according to $mz'' = -mg$ where g is the gravitational constant. Consider the effects of adding a small nonlinear frictional resistance. The frictional force opposes the motion,

$$mz'' = -mg - \varepsilon\, z'|z'|\,, \qquad z(0) = h > 0\,, \quad z'(0) = 0\,, \qquad 0 < \varepsilon << 1\,.$$

Compute the leading order correction. That is compute an approximate solution whose leading term is the unperturbed solution and the correction term is the first corrector in perturbation theory that is not identically equal to zero.

Discussion. This is an example of a practical problem for which the nonlinear function is only differentiable a finite number of times.

3-4. 37/2a,c.

5. 37/3a,c. That is, identify the phase portraits of the systems solved in the preceding exercise.

6. 37/4. **Hint.** You can reduce to the case $a = 0$ and save some effort.

7. 58/4. For parts (b) and (c) only discuss those cases that do **not** have repeated eigenvalues. **Remark.** This is the same as 38/6 but stated more clearly. **Discussion.** After we discuss repeated eigenvalues you might want to have a second look at this question to treat the omitted cases.