Math 558 Fall 2014 — Prof. J. Rauch

Homework 4 — Due Thursday 2 October 2014

1. 57/1.

2. 57/1 Addendum 1. This question concerns phase portrait number 1.

i. Compute the direction of the major and minor axes of the ellipses.

ii. Compute the aspect ratio of the ellipse, that is, the length of the major axis divided by the length of the minor axis.

iii. Find a quadratic conserved quantity for the differential equation.

3. 57/1 Addendum 2. Consider the equation

$$X' = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{\dagger}$$

with real constants a, b, c, d. Briefly explain each answer.

i. Find the equation whose phase portrait is the reflection in the x_1-axis of the phase portrait of system (†).

ii. Same question for the x_2-axis. You need not repeat the explanation from **i**.

iii. Find the equation whose phase portrait is the same as that of system (†) but with the arrows reversed.

iv. What is the relation between the matrices leading to the phase portraits (1) and (4) of 57/1?

4. 58/2 Variant. For the system

$$X' = \begin{pmatrix} 1 & 1 \\ -2 & 0 \end{pmatrix} X .$$

Answer questions (a) and (d) and the $X' = AX$ part of (c).

5. For the system

$$X' = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix} X .$$

perform the computations (a) to (d) from 58/2 and

(e) Compute the invariant lines in $\mathbb{R}^2$ and label as stable and unstable on the sketch.

(f) Compute a nontrivial continuous conserved quantity if one exists or show nonexistence.

Continued on the next page.

6. Use the method of finding a exponential series that terminates after a finite number of terms to find the general solution of the following system

$$X' = \begin{pmatrix} 3 & 4 \\ -1 & -1 \end{pmatrix} X\,.$$

Discussion. The matrix does not have a basis of eigenvectors.

7. Nonlinear friction continued. Consider free fall with nonlinear friction as problem Homework 3. Then the velocity $v := z'$ satisfies the equation

$$mv' = -mg - \varepsilon v|v|\,, \qquad 0 < \varepsilon\,.$$

i. Sketch a graph of the function $f(v) = -mg - \varepsilon\, v|v|$.

ii. Perform a phase line analysis of this equation. Determine the asymptotic behavior as $t \to +\infty$ of $v(t)$. Describe the conclusion in words. Explain why it is intuitively reasonable.

Discussion. A final step in this problem will appear on the next assignment.