Math 558 Fall 2014 | Prof. J. Rauch
Homework 5 | Due Thursday October 9

1. Nonlinear friction continued. Consider free fall with nonlinear friction as in Homeworks 3,4.

i. Find the asymptotic behavior as $t \to \infty$ of the approximate solution found in Homework 3.

Discussion. This divergence to infinity is called secular growth.

ii. Does the asymptotic behavior of the approximate solution predict that of the exact solution as computed in Homework 4.

Discussion. The approximate solution is guaranteed to be a good approximation on bounded time intervals.

iii. Show that $\lim_{\varepsilon \to 0} \lim_{t \to \infty} v_{\text{approx}} \neq \lim_{t \to \infty} \lim_{\varepsilon \to 0} v_{\text{approx}}$.

Discussion. i. It is normal that an approximation has small errors. Those errors accumulate over time. It is rare that an approximate solution gives a good approximation for very large times. **ii.** Concerning the third part V. Arnold offers the excellent example of a bucket full of water with a hole in the bottom of size ε. For that system, $\lim_{\varepsilon \to 0} \lim_{t \to \infty}$ is an empty bucket while $\lim_{t \to \infty} \lim_{\varepsilon \to 0}$ is a full bucket.

2. Exercise 1.1 of the Spectral Decomposition Handout.

3. Show that if the 2×2 matrix A has only one eigenvalue $\underline{\lambda}$ and there is a basis of eigenvectors, then $A = \underline{\lambda}\, I$.

4. 71/1.

The next problems use the Spectral Decomposition Theorem to find the general solution of $X' = AX$ when A need not have a basis of eigenvectors. The method is succinctly described in the Multiple Root Algorithm handout.

5. 135/1a.

6. 135/5.

7. 135/1c.