Math 558 Fall 2014 — Prof. J. Rauch

Homework 9 — Due Tuesday November 11

1. Exercise 1.2 in the Spectral Theory handout.

2. For each of the following 2×2 matrices A, find a positive definite quadratic form Q that decreases on orbits of the system, $X' = AX$.

$$\textbf{i.}\ \begin{pmatrix} -2 & 0 \\ 0 & -3 \end{pmatrix}, \qquad \textbf{ii.}\ \begin{pmatrix} -1 & 1 \\ -4 & -1 \end{pmatrix}.$$

3. The nonlinear system $X' = F(X)$ has an equilibrium at $X = 0$ and $F_X(0) = A$ the matrix from the example on page 112-113 of Hirsch-Smale-Devaney.

i. Find the dimensions of the stable and the unstable manifolds at 0.

ii. Find the tangent vector spaces to the stable and to the unstable manifolds at 0. **Hint.** Use problem 1.

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5. i. Find all equilibrium solutions of the following system and determine, where possible, their stability by linearization,

$$x' = x^2 + y^2 - 1, \qquad y' = x^2 - y^2 .$$

ii. For each equilibrium that is a saddle, find the tangent to the stable and unstable manifolds at the equilibrium.

6. i. Show that

$$x' = \ln(1-z), \qquad y' = \ln(1-x), \qquad z' = \ln(1-y),$$

has only one equilibrium and that it is hyperbolic (no purely imaginary eigenvalues for the linearization).

ii. Find the dimensions of the stable and unstable manifolds. Find the tangent space at the equilibrium of the stable and unstable manifold. **Hint.** Use problem 1.

7. Determine the stability of the equilibrium 0 of the following system from its linearization,

$$\begin{aligned} x_1' &= -2x_1 + x_2 + 3x_3 + 9x_2^3 \\ x_2' &= -6x_2 - 5x_3 + 7x_3^5 \\ x_3' &= -x_3 + x_1^2 + x_2^2 . \end{aligned}$$