Math 558. Advanced Ordinary Differential Equations and Dynamical Systems
Fall 2014
Prof. J. Rauch

Prerequisites. Basic Linear Algebra, Ordinary Differential Equations (Math 216), Multivariable Calculus (215). Some exposure to more advanced mathematics *e.g.* Advanced Calculus (Math 450/451) or Advanced Mathematical Methods (Math 454).

Text. M. Hirsh, S. Smale, and R. Devaney, *Differential Equations, Dynamical Systems, and an Introduction to Chaos, 3rd ed.*, Elsevier. The UM library has an electronic version which can be freely consulted as many times as you like. Many supplementary materials posted on my homepage.

Homework. Graded assignments weakly.

Exams. In class midterm plus final exam.

Course Description.

Differential equations model systems throughout science and engineering and display rich dynamical behavior. This course emphasizes the qualitative and geometric ideas which characterize the post Poincaré era. The course surveys a broad range of topics with emphasis on techniques, and results that are useful in applications. It is intended for students in mathematics, engineering, and the natural sciences and is a core course for the Applied and Interdisciplinary Mathematics graduate program. Proofs are given. Homeworks and exams concentrate on using rather than proving.

Outline. Chapters 1-10 + Ch. 15 of Hirsh-Smale-Devaney. Plus online materials prepared to complement the text. There are more complements than we will treat.

- Phase line. Dynamics in dimension 1 and 1.5. Bifurcations. Poincaré map.
- Existence, uniqueness, perturbation theory.
- Linearization at equilibria. Theory of constant coefficient systems. Spectral theorems.
- The geometry of phase plane of linear systems..
- Stable and unstable manifolds.
- Conjugation of sinks/sources.
- Lyapunov's method. LaSalle's invariance principal.
- Gradient flows and hamiltonian systems.
- Periodic solutions, stability, Poincaré map, ω-limit set, Poincaré-Bendixson (time permitting).
- Bifurcation theory of equilibria. Pitchfork and Hopf.
- Introduction to chaotic dynamics. Definitions and first examples.

Grading. Homework 35%, Midterm Exam 25%, Final Exam 40%.