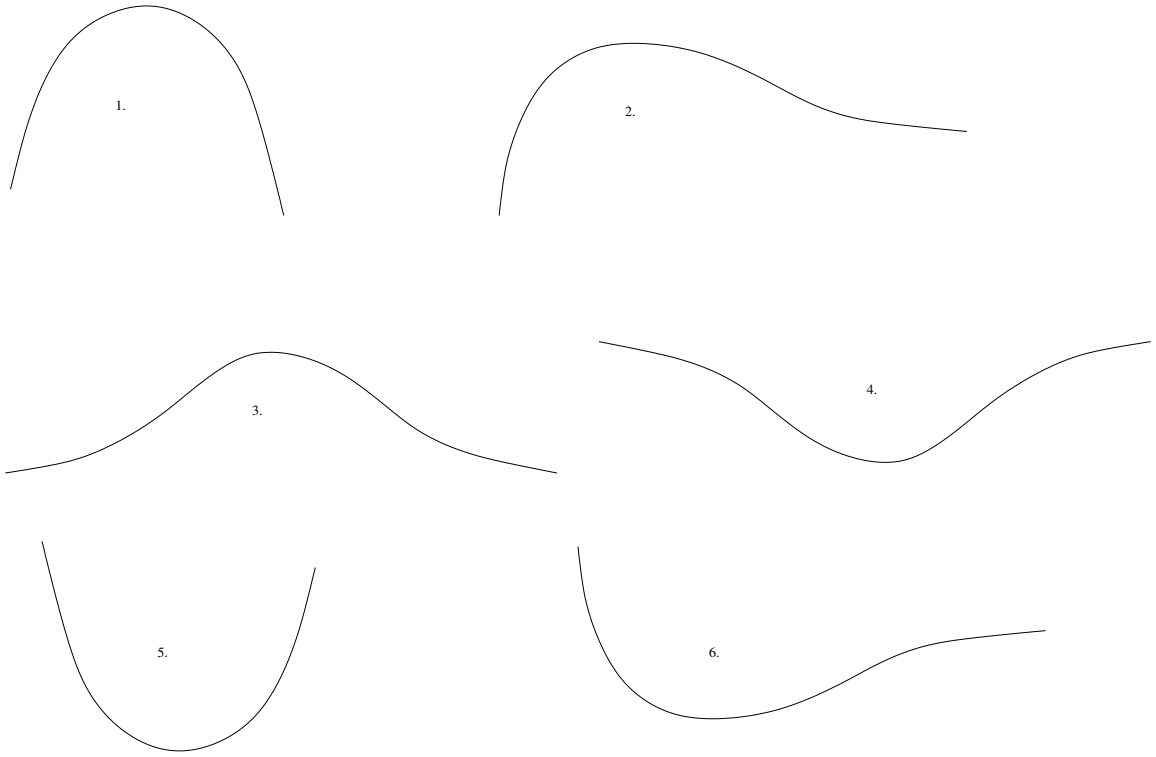
J. Rauch Math 558

Problems for $1-d$ Mechanical Systems

1 Potentials with one critical point.

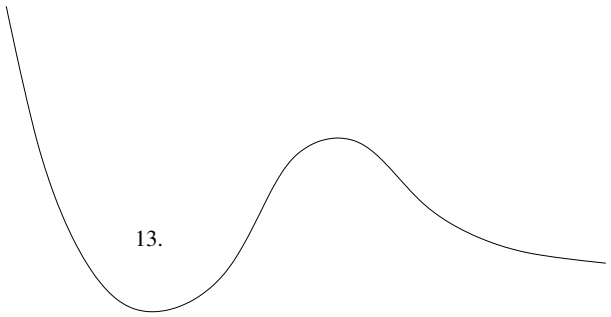
Each of the following six graphs is the graph of a potential $V(x)$ with exactly one critical point. You may assume that $V'' \neq 0$ at the critical points. You may also assume that for large x either V tends to infinity or a horizontal asymptote as indicated in the figure. In each case you are asked to sketch the entire phase portrait of the mechanical system

$$x'' \;=\; -\frac{\partial V(x)}{\partial x}\,.$$

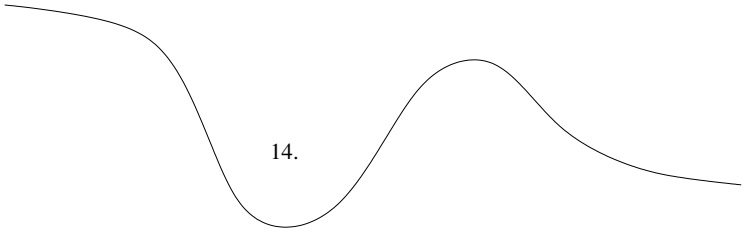


2 Potentials with two critical points.

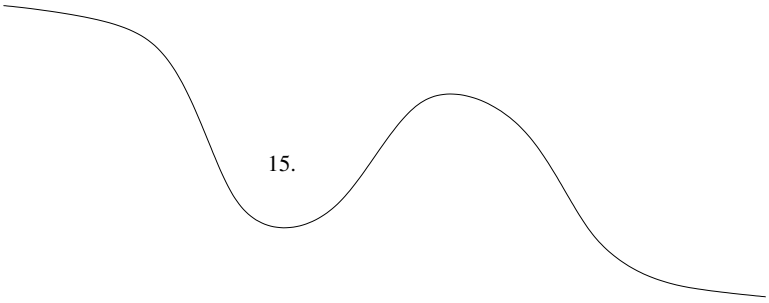
Perform the same analysis for the following potentials each of which has two cricital points.



13.



14.



15.

3 Potentials with three critical points.

Perform the same analysis for the following potentials each of which has three critical points.