Mathematics 571
Winter 1999
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Assignment #1. Due 14 January.

1. Let A denote an $N \times N$ matrix and $\mathbf{e} := (1, 0, \ldots, 0)$ the first elementary basis vector of $\mathbb{R}^N$. Find the relation between the vectors $A\mathbf{e}$, $A^T\mathbf{e}$, and the matrix A.

2. a.) Show that a permutation matrix is orthogonal. b.) Give an example of a 4×4 permutation matrix whose eigenvalues are the four complex numbers $\pm 1, \pm i$. **Discussion.** In particular this shows that the eigenvalues of an orthogonal matrix are not necessarily real.

3. Ciarlet problem 1.1-5 on page 8. In future we will use the shorthand 8/1.1-5.

4. Consider the 2-point boundary value problem

$$-\frac{d^2\phi(x)}{dx^2} + x\,\phi(x) = (1 + 2x - x^2)\,e^x\,, \quad \text{for} \qquad 0 \le x \le 1\,,$$

$$\phi(0) = 1, \quad \phi(1) = 0\,.$$

Verify that $\phi(x) = (1-x)e^x$ is a solution. We will later show that this is the only solution. This is a consequence of the fact that the coefficient of ϕ which Ciarlet calls $c(x)$ is in this case equal to x which is nonnegative on the interval $[0, 1]$.

Write a computer program to solve the problem using the second order finite difference scheme discussed in class and in §3.1 of Ciarlet. Solve the resulting tridiagonal system using the LU factorization. Run the code for $h = 2^{-p}$, $p = 1, 2, \ldots, 12$. Display the results in the same format as Table 3.1-1 on page 74 of Ciarlet's book. Discuss the behavior of the error as h decreases. **Discussion.** You should discern the order of accuracy of the method and something else too.