Mathematics 571 Winter 1999
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Assignment #10. Due April 1.

1. (Ciarlet 1.5-1). Show that every bounded sequence of unitary matrices has a convergent subsequence. Show that the limit of each such subsequence is unitary.

2. (Golub and VanLoan 8.1-4). Compute the Schur decomposition (Thm 1.2-1 part (1)) of the matrix.

$$B \;=\; \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}.$$

3. (Conte-DeBoor 4.8-11). Find the eigenvalues of the matrix

$$A \;=\; \begin{pmatrix} 4 & -1 & -1 & -1 \\ -1 & 4 & -1 & -1 \\ -1 & -1 & 4 & -1 \\ -1 & -1 & -1 & 4 \end{pmatrix}$$

by determining explicitly its charateristic polynomial and then finding the roots. This is an exact computation.

4. Find the eigenvalues of the matrix A from problem 3 by the classical Jacobi algorithm. You should write your own code, and plot error versus iteration number to get a feel for the efficiency of this method.

Since there is a multiple eigenvalue, Theorem 6.1-3 does not apply. For your computation do you observe that the orthogonal matrices

$$\Omega_1\,\Omega_2 \ldots \Omega_{k-1}\,\Omega_k$$

converge as k increases? If so, what eigenvectors do they give?

Discussion. Let $N := n(n-1)/2$ be the number of off diagonal elements. By a sweep we mean N iterations of the basic step. Set $\epsilon_k := \sum_{i\neq j} |a^k_{ij}|^2$. The convergence is measured by the rate that ϵ_k tends to zero. In class we proved linear convergence,

$$\epsilon_{k+N} \le \Big(1 - \frac{1}{N}\Big)^N \epsilon_k \approx \; \frac{1}{e}\,\epsilon_k$$

during each sweep. It is known (see Golub and Van Loan §8.4 for references) that for k large one has quadratic convergence, that is

$$\epsilon_{k+N} \le c\,{\epsilon_k}^2\,.$$

In linear convergence you gain 1.5 binary digits per sweep. With quadratic convergence the number of significant digits doubles during each sweep. You have seen quadratic convergence in Newton's method.