Mathematics 571
Winter 1999
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**Assignment #4. Due Tuesday February 8.**

Note. Class will **not** meet Thursday February 4. The HW is due on the following Tuesday. The class will be made up later in the term.

**1.** Prove that for all $x \in \mathbb{C}^n$,

$$\|x\|_\infty \le \|x\|_2 \le \|x\|_1 \,.$$

**2.** Find the smallest constant $C(n)$ so that for all $x \in \mathbb{C}^n$

$$\|x\|_1 \le C(n)\|x\|_\infty \,.$$

You must prove the estimates and show also that it is best possible.

**3.** Find the smallest constant $D(n)$ so that for all $x \in \mathbb{C}^n$

$$\|x|_1 \le D(n)\|x\|_2 \,.$$

You must prove the estimates and show also that it is best possible.

**4.** Show that if $|\gamma| < \|A\|^{-1}$ for a subordinate matrix norm $\| \ \|$ then $I + \gamma A$ is invertible. Find the first three terms in a convergent power series expansion

$$(I + \gamma A)^{-1} = B_0 + \gamma B_1 + \gamma^2 B_2 + \dots \ .$$

**5.** Let

$$A := \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} .$$

The singular values of $A$ were computed in class. Compute directly the singular values of $A^*$ and verify that they are equal to the singular values of $A$.

**Extra credit.** Is it true that the singular values of a matrix and its adjoint are always equal? Find a proof or a counterexample.