Mathematics 571
Winter 1999
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Assignment #6. Due February 23.

1. If A has the LU factorization $A = LU$ is it true that the diagonal elements of U are the eigenvalues of A. Give proof or counterexample.

2. In class we showed that if a and b distinct vectors in $\mathbb{R}^n$ with $\|a\|_2 = \|b\|_2$ and $u := (a - b)/\|a - b\|_2$ then the Householder matirx $H = H(u)$ satisfies $Ha = b$. This is one half of Problem 4.5-2. Complete that exercise by showing that there is only one Householder matrix with $Ha = b$. That is, show that if $Ha = b$ then $H = H(u)$ with u as above. **Discussion.** In class we agreed that this assertion is very reasonable given the geometric interpretation of Householder matrices.

3. Let $\phi(x_1, x_2)$ be the steady state temperature in a square plate heated on one side and cooled on the other three. Then ϕ is the solution of the Dirichlet problem

$$\Delta\,\phi = 0 \qquad \text{for} \qquad (x_1, x_2) \in]0,1[\times]0,1[\,,$$

$$\phi(x_1, 0) = 1\,, \qquad \phi(0, x_2) = \phi(1, x_2) = \phi(x_1, 1) = 0\,.$$

Compute approximate solutions using the five point Laplacian with mesh size $h = 2^{-p}$, $p = 2, 3, 4, 5$. Solve the linear system by Cholesky factorization. Use a contour or surface plotting routine (e.g. in matlab) to plot the numerical solutions. Include the boundary values in the plotted values.