Mathematics 571
Winter 1999
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Assignment #7. Due March 11.

1. Show that there is only one square matrix A such that the powers A^k converge to an invertible matrix. This is Ciarlet problem 1.5-2. **Hints.** Shur's theorem shows that $A = M^{-1}(D+U)M$ with unitary M, diagonal D and strictly upper triangular U. Show that A^k converges if and only if $(D+U)^k$ converges. Show next that if $(D+U)^k$ converges then $D = I$. Finally show that if $(I+U)^k$ converges then $U = 0$.

2. Part (2) of Ciarlet problem 4.5-1.

3. Ciarlet problem 2.2-2.

4. Let $\phi(x_1, x_2)$ be the steady state temperature in a square plate heated on one side and cooled on the other three. Then ϕ is the solution of the Dirichlet problem

$$\Delta \phi = 0 \qquad \text{for} \qquad (x_1, x_2) \in]0,1[\times]0,1[\,,$$

$$\phi(x_1, 0) = 1\,, \qquad \phi(0, x_2) = \phi(1, x_2) = \phi(x_1, 1) = 0\,.$$

Compute approximate solutions using the five point Laplacian with mesh size $h = 2^{-p}$, $p = 4, 5, 6$. *Solve the linear system by Jacobi iteration and by Gauss-Seidel iteration.* Write your code so that you have confidence that the computed solution has error at most 1%. Compare the number of iterations needed to reach this precision and discuss the relation with the theorems of Chapter 5. **Hint.** You have experience from last time in constructing the matrix A. However in this sparse setting is is wasteful to find the matrix A and wiser to set up the iteration using the few nonzero elements in A. You are permitted to be wasteful but think a little about how you would start from scratch using the sparseness to advantage.