Mathematics 571
Winter 1999
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Assignment #8. Due March 18.

1. Ciarlet 5.2.3. This result asserts that if A is strictly diagonally dominant then the point Jacobi and point Gauss-Seidel methods converge. **Hints.** One method is to show that a subordinate norm of the iteration matrices is strictly less than one. Gauss-Seidel is trickier than Jacobi. The argument resembles the proof of the Gershgorin Theorem. A second option is to to show that if $|\lambda| \geq 1$, then $\det(B - \lambda I) \neq 0$. The manipulations of the determinant resemble arguments we have seen before.

2. Consider the equation $Ax = b$ with

$$A := \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} . \qquad b := \begin{pmatrix} 5 \\ 5 \end{pmatrix} .$$

Take two steps of both Jacobi and Gauss-Seidel methods starting from $x^0 := 0$. Compute the iteration matrices B_J and B_{GS} and their spectral radii.

3. Consider again the boundary value problem from assignment #7, problem 4. For $h = 2^{-p}$, $p = 3, 4, 5$ use Block Jacobi and Block Gauss-Seidel to solve the resulting linear system starting from the initial guess $x^0 := 0$. Use the same accuracy criterion as in assignment #7. From the output estimate the the spectral radius of the iteration matrices for these examples. Compare the rate of convergence with the point Jacobi and point Gauss-Seidel methods employed in assignment #7. **Hints.** If you are using Matlab, avoid loops and use as much as possible the built in matrix language of Matlab. If you take advantage of the sparseness, these computations go quickly. If not, and if you write loops in Matlab they can go VERY slowly. In performing the iterations you will need to solve equations $Dx = y$ where D is one of the diagonal blocks. These will be tridiagonal so can be very rapidly solved by tridiagonal Cholesky.

4. Using the estimates from the last problem and Theorem 5.3-6, estimate the optimal relaxation parameter and the spectral radius of the resulting optimal Block SOR method. Using this result, estimate how many iterations of optimal SOR would be required to achieve the same accuracy as required in the previous problem for $p = 3, 4, 5$. **Discussion.** It is interesting to note that since the clean relation between the spectral radii which we have proved is valid only in the block tridiagonal setting, one could not append a problem analogous to this one for the point Jacobi and SOR methods applied to this A since A is not tridiagonal. The point methods were employed in assignment #7.