Help with the proof of Theorem 6.2-3.

The proof in the last ten lines of page 202 of Ciarlet and the version which I presented in class today both strike me as too rapid. The hint to look at the Figure in the text is well taken. Here is a little help if you are trying to struggle through the details.

A key is to consider the sign of $p_i(x)$ as x increases. For x very negative it is positive. It crosses to negative at the first root, then to negative at the second and so on. Thus for any i and for odd (resp. even) N, p_i is negative (resp. positive) between λ^i_N and λ^i_{N+1}.

Thus if $\mu < \lambda^{i+1}_{N(i,\mu)+1}$ then one has

$$\mu \ \in \,]\lambda^i_{N(i,\mu)} \, , \, \lambda^i_{N(i,\mu)+1}[\, ,$$

AND

$$\mu \ \in \,]\lambda^{i+1}_{N(i,\mu)} \, , \, \lambda^{i+1}_{N(i,\mu)+1}[\, .$$

Therefore $\operatorname{sgn} p_i(\mu) = \operatorname{sgn} p_{i+1}(\mu)$ is determined by the parity of $N(i,\mu)$. This justifies the implication on line -8 of page 202.

On the other hand if $\mu > \lambda^{i+1}_{N(i,\mu)+1}$ then

$$\mu \ \in \,]\lambda^{i+1}_{N(i,\mu)} + 1 \, , \, \lambda^{i+1}_{N(i,\mu)+2}[$$

and the signs of $p_i(\mu)$ and $p_i(\mu+1)$ are opposed.

This justifies line -6 of the text. The case of equality in line -4 is left to the reader.