Autour de Guy, et,

Focalisation Optimale Electromagnétique

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Colloque Chocs et Oscillations

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Focussing preprint on my home page and on `arxiv`.

Outline

Part 1. A biography in photos.

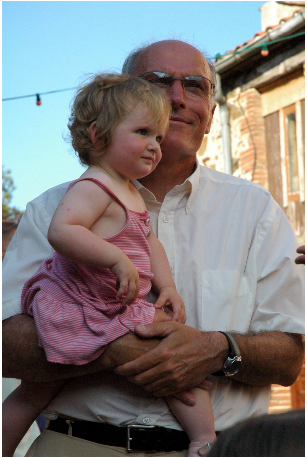
Part 2. A retrospective of Guy's mathematics.

Part 3. Optimal focusing.

frere Jean et Guy,

Guy et Simone

I. Simone, Guy, Raphael, Florence, Elizabeth et Guy



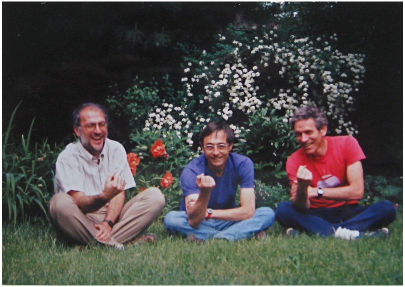
Some Guy passions

From the JMR archives. Paris garret appartment 1992.

Ann Arbor barbeque 1992.

Chez moi 1992, Lemma hunt, Lake Michigan 1994.

Bras de fer, Ann Arbor 1992, Bordeaux 2005.



Six slides of Guy math. First steps, spectral theory

Self adjoint eigenvalue counting in non traditional cases.

Hörmander's sum of squares. Interior term not determined by principal symbol.

d-bar Neumann operator a prime example.

Irregular boundary, and/or ellipticity degenerates. Changes the analysis on a neighborhood of the boundary.

First appearance of physics.

Series of articles with Alinhac ($\sim$ 1983-86) on local propagation of analyticity.

Includes the *incompressible Euler equations.* Compressible is easier because the incompressible case has the *nonlocal* projection on divergence free fields.

Analyticity tools resurface 15 years later in "Dense Oscillations for the Compressible Euler Equations II". The leading profile is analytic.

Early shocks (mid to late eighties).

Series of papers on propagation, reflection, and interaction of conormal waves, sometimes with M. Beals or J. Rauch.

Warmup for existence and interaction (in the 2×2 genuinely nonlinear case) of multidimensional shocks.

Bare hands. Nash-Moser no. Paradifferential operators yes.

And the weak shock limit (ondes soniques).

Strong shocks are noncharacteristic. Weak shocks are nearly characteristic posing difficulties. Singularities of conormal waves are characteristic too.

Oscillations

JMR had a great time studying oscillations, with electromagnetism, and fluids always on our minds.

Don't forget Guy's work on supercritical oscillations with Guès and Cheverry.

Zumbrun et. al.

Following earlier work of Gisclon-Serre, Zumbrun and different *et. al.*, Grenier-Guès, ... *etc.*, Guès, Métiver, Williams, and Zumbrun revolutionized our understanding of the stability and inviscid limit for *multidimensional* boundary layers and shock layers.

The stability criteria they impose rather than smallness, involves Evans' functions and *are physically natural* and *necessary*.

Both layers and oscillations use multiscale expansions with correctors. Introduce a variety of new technique.

Lessons. **1.** *leading term asymptotics are almost never enough to get error control*. **2.** *Two scale expansions beat matching for proofs.*

Four more.

Incompressible limit with Schochet. A singular limit that is a cousin to geometric optics. (Also a variety of large Coriolis limits in geophysics).

Laser-Plasma interaction with Colin. Treatise: "Mathematics of Nonlinear Optics".

Clarification of "bloch structure" in Kreiss type symmetrizer constructions at glancing.

Nonlinear scalar Hölmgren Theorem is a return to origins. With counterexamples in the nonscalar case.

My math talk: Optimal focussing.

Consider solutions of Maxwell's equations,

$$E_t = \mathrm{curl}\, B, \quad B_t = -\mathrm{curl}\, E\,, \quad \mathrm{div}\, E = \mathrm{div}\, B = 0\,,$$

that are **monochromatic**,

$$\psi(t)\,\big(E(x)\,,\,B(x)\big)\,, \qquad \text{with} \qquad \psi'' + \omega^2\,\psi \;=\; 0\,, \quad \omega \;>\; 0\,.$$

Problems (from G. Mourou/ELI). **1.** *For fixed incoming energy, how much energy can be focussed into a small ball?* Say radius equal to 1/10 of a wavelength.

2. *How large can the electric field at the origin be?*

Preparing the solution.

Scaling reduces to the case $\omega = 1$, wavelength $= 2\pi$. In that case *all have the form*,

$$\mathrm{E}(\mathrm{x}) \;:=\; \int_{|\xi|=1} \mathrm{e}^{\mathrm{i}\mathrm{x}\xi}\, \mathbf{e}(\xi)\; \mathrm{d}\sigma\,, \quad \mathbf{e} \;\in\; \mathcal{D}'(\mathrm{S}^2), \quad \mathbf{e}\cdot\xi = 0\,.$$

The solutions with $\mathbf{e} \in \mathrm{L}^2(\mathrm{S}^2)$ *are the ones of interest.*

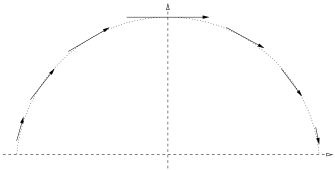
• Have fastest, $\mathrm{O}(\mathrm{r}^{-1})$, decay.
• In $\mathrm{r} >> 1$ are the sum of an incoming and outgoing wave each with $\mathrm{Energy}(\mathrm{R} \le |\mathrm{x}| \le 2\mathrm{R}) \sim \mathrm{R}\,\|\mathbf{e}\|_{\mathrm{L}^2}$. So $\|\mathbf{e}\|$ fixes the incoming energy.
• Yield spectral projectors.

Definition of the longitude vector field $\ell(\xi)$.

On $|\xi| = 1$, $\ell(\xi)$ is the projection of the vector $(1, 0, 0)$ orthogonal to ξ (equivalently tangent to $\mathrm{S}^2 := \{|\xi| = 1\}$),

$$\ell(\xi) \ := \ (1,0,0) \ - \ \big(\xi \cdot (1,0,0)\big)\, \xi \ = \ (1,0,0) \ - \ \xi_1\, \xi \,.$$

The field ℓ is obtained by rotating the figure about the horizontal axis.



ℓ is the gradient of the function ξ_1 defined on S^2.

Theorem. *If* $\mathbf{e} \in L^2$ *and* E *is the corresponding monochromatic solution, then*

$$\|E(0)\| \;\le\; \left(\frac{2}{3}\,|S^2|\right)^{1/2}\,\|\mathbf{e}\|_{L^2(S^2)}\,, \qquad dim = d, \quad \frac{d-1}{d}\,|S^{d-1}|$$

with equality if and only if $\mathbf{e}$ *belongs to the three dimensional space of of scalar multiples of rotates of* ℓ*.*

Proof. *i.* Maximizing $\|E(0)\|$ is equivalent, by rotational invariance, to maximizing the first component

$$E_1(0) \;:=\; \int_{|\xi|=1} \mathbf{e}(\xi)\cdot(1,0,0)\; d\sigma\,.$$

ii. Since $\mathbf{e}\cdot\xi = 0$, $\mathbf{e}\cdot(1,0,0) = \mathbf{e}\cdot\ell$. *iii.* By Cauchy-Schwartz,

$$|E_1(0)| \;=\; \left|\int \mathbf{e}(\xi)\cdot\ell(\xi)\; d\sigma\right| \;\le \|\ell\|\,\|\mathbf{e}\| \;=\; \left(\frac{2}{3}\,|S^2|\right)^{1/2}\,\|\mathbf{e}\|\,,$$

with equality if and only if $\mathbf{e}$ is a scalar multiple of ℓ. □

The optimal monochromatic field.

Solves the problem of maximizing the field at a point.

The max is $\sqrt{2/3}$ times the max for scalar waves.

I am surprised not to find the solution in the literature. If you know a reference please tell me.

The same field delivers the largest energy to balls of radius $R < R_0 \approx wavelength/3$. Not for $R > R_0$.

Both the outgoing and incoming waves making up the optimizer have leading far field profile proportional to $\ell(x/|x|)/|x|$.

Summary. *To achieve maximal concentration of energy, generate an incoming wave in the far field with leading profile* $\ell(x/|x|)$.

Thank you for your attention, and, best of everything to Guy. Santé!