Boundary Value Problems for Partial Differential Equations

Course proposal to INDAM from Jeffrey RAUCH

This two month course is intended for graduate students in mathematics. As background it requires Calculus of Several Variables, Elementary Complex Analysis, the rudiments of Functional Analysis up to duality in Hilbert space, and a good first course in Partial Differential Equations.

Two topics will be discussed. The first is recent advances in Friedrich's theory of Symmetric Positive Boundary Value Problems. This theory dating to the 1940s was originally aimed at providing a setting appropriate for problems of mixed hyperbolic-elliptic type. In the course of time, it has served some in that direction but more importantly as a unifying form for many problems in mathematical physics.

Recently, responding to needs in numerical analysis, the subject has been revisited in the setting of domains in corners, most importantly rectangles in $\mathbb{R}^d$. For most problems the boundary conditions on adjacent faces are necessarily different. The simplest example is the Neumann problem for the Laplace equation where the normal direction is discontinuous from face to face. A related problem is domains with smooth boundaries but with boundary conditions that are discontinuous. Following Friedrichs we study problem that have an easy *a priori* estimate so uniqueness of smooth solutions and existence of weak solutions is automatic. The problem is the gap between smooth and weak. The goal is to find classes with existence **and** uniqueness. Two natural approaches are to prove existence of more regular solutions and/or uniqueness of less regular solutions. Progress has been made in both directions. The course will present the classic setting of smooth boundaries and smooth boundary conditions and then turn to recent progress and challenges that remain.

The second topic is even older, the use of complex variables to find steady incompressible irrotational two dimensional fluid flows. This is a standard topic in complex analysis and is described in many texts. However, there are uniqueness questions that are needed to justify the application of these solutions in physical problems. Virtually all the solutions proposed in texts are not unique without further restrictions. In this course we describe two things. First that the non uniqueness is proved by explicit complex variables formulas. And second, that the uniqueness questions too can be analysed by elementary complex variable techniques.