# Meet the Laplacian. Spherical Symmetry and the Mean Value Property

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## Outline

- Rotational invariance of $\Delta$ and harmonic functions.
- Symmetry derivation of MVP
- The Laplacian as Infinitesimal MVP
- Equal angles of isoceles triangle, symmetry proof from Euclid's elements
- Coulomb's Law and Gauss' Law
- Symmetry proof of Newton's Theorem
- Proof of Newton's Theorem by MVP
- The five point Laplacian
- Discrete Dirichlet Problem
- Random walk and the discrete Dirichlet problem
- Random walk and the Dirichlet problem.

The Coulomb, Gauss, Newton trilogy is an introduction to Electrostatics and prepares the Meet Electrostatics topics.

**Course Materials**. There is a handout on the Infinitesimal MVP. There a handout on Haar measure. One on Coulomb, Gauss, and Newton. The last three topics are inspred by the classic paper of Courant, Friedrichs, and Lewy, a translation of which is posted on my home page. Click to course materials and scroll to Encounters with Partial Differential Equations.

A set of **Excercises** on the laplacian and symmetry is also my home page. It contains yet another proof of the Mean Value Property that requires identifying all orthogonally invariant polynomials.