MATH 451 HOMOWORK SET 12 (ADDITIONAL)

1. Let
$$f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & \text{for } x \neq 0; \\ 0, & \text{for } x = 0. \end{cases}$$
Show that f is differentiable on $\mathbb{R}$, and f' is not continuous on $\mathbb{R}$.

2. (optional) True or False: Assume that f is a differentiable function on (a, b). Then for any $\xi \in (a, b)$, there are $x_1, x_2 \in (a, b)$, such that $x_1 < \xi < x_2$, and
$$f'(\xi) = \frac{f(x_1) - f(x_2)}{x_1 - x_2}.$$
Hint: Consider $f(x) = x^3$, for $x \in (-1, 1)$. Draw the graph.