Lie groups in $\mathrm{GL}_n(\mathbb{R})$

Problem 23. Suppose that $\gamma(t)$ is smooth curve in GL_n with $\gamma(0) = 0$ and $\gamma'(0) = X$. Show that $\lim_{n\to\infty} \gamma(1/n)^n = \exp(X)$.

Let G be a subgroup of $\mathrm{GL}_n(\mathbb{R})$ which is also a manifold in $\mathrm{Mat}_{n\times n}(\mathbb{R}) = \mathbb{R}^{n^2}$. We should also have said **Assume further that G is closed in $GL_n(\mathbb{R})$.** Let $\mathfrak{g}$ be the tangent space $T_e G$.

Problem 24. Show that $\exp(\mathfrak{g}) \subseteq G$.

Problem 25. Show that, if X and $Y \in \mathfrak{g}$, then $XY - YX \in \mathfrak{g}$. Hint: Consider $e^{sX}e^{tY}e^{-sX}e^{-tY}$.

Problem 26. Show that there are open sets $U \ni e$ and $P \subset \mathfrak{g}$ such that $\exp : P \to G \cap U$ is bijective. (This is hard! I think I would choose a subspace W of $\mathrm{Mat}_{n\times n}(\mathbb{R})$ complementary to $\mathfrak{g}$ and think about condition (4).)