The orthogonal group as a manifold

Let O(n) be the group of $n \times n$ real matrices X with $XX^T = \mathrm{Id}_n$.

Problem 16. Show that there is an open set $U \ni Id_n$ such that $U \cap O(n)$ is a manifold.

Problem 17. Compute $T_{\mathrm{Id}_n}O(n)$.

Problem 18. Show that O(n) is a manifold. (Hint: Use that O(n) is a group!)