WHAT WE'VE LEARNED THIS YEAR

The exam will concentrate most heavily on Chapters 7 and 8 but will cover the entire term. All of the following may appear on the exam.

Chapter 1 – Solving linear equations

- Reduced row echelon form
- Using row reduction to solve linear equations

Chapter 2 – Matrices and linear maps

- Basic matrix operations – addition, multiplication, inverse, transpose.
- Injective, surjective, invertible. What these terms mean? How do you test them?
- Using matrices to describe geometry – rotations, reflections, projections, etc.
- Computing the inverse of a matrix by row reduction.

Chapter 3 – Subspaces and dimension

- Definition of a subspace of $\mathbb{R}^n$
- Linearly independent, spanning, basis. What do these words mean? How do you test them?
- Dimension of a subspace.
- Computing a basis for the image or kernel of a linear map.

Chapter 5 – Dot product

- How to compute $\vec{u} \cdot \vec{v}$.
- Geometric interpretation of $\vec{u} \cdot \vec{v}$. Computing lengths of vectors and angles between them.
- What does it mean for a list of vectors to be orthonormal?
- What does it mean for a matrix to be orthogonal?
- What is an orthogonal basis for a subspace? How do you compute it?
- Orthogonal projection – what does it mean and how do you compute it?
- Least squares approximation

Chapter 6 – Determinants

- When is $\det A = 0$, when is $\det A \neq 0$?
- How do operations like adding rows, interchanging rows, rescaling, taking inverse and transpose and multiplying matrices affect determinants?
- Using the properties above to calculate determinants
- Using row reduction to calculate determinants
- Using the sum of patterns formula to calculate determinants
- Geometric meaning of determinant

Chapter 7 – Eigenvalues and Eigenvectors

- What is an eigenvalue? What is an eigenvector? What is the characteristic polynomial?
- How do we compute the characteristic polynomial?
- How do we find eigenvalues?
- How do we find eigenvectors?
- Using eigenvalues to approximate $A^n\vec{v}$ for n large.
- What does it mean to diagonalize a matrix? How do you do it?
- When the characteristic polynomial has complex roots, how do you compute the behavior of A^n? What does it look like geometrically?

Chapter 8 – Quadratic forms

- What is the relation between quadratic forms and symmetric matrices?
- What does $x^2 + 2y^2 - 3z^2 = 1$ look like (and other equations like this)?
- What can you say about the eigenvalues of a symmetric matrix? The eigenvectors?
- How can you put a quadratic matrix into diagonal form?
- How do you compute the singular values of a matrix?
- What do singular values say about the geometry of a linear map?