The field axioms

Let F be some set of numbers with operations of addition and multiplication.Then F is called a ***field*** if it obeys the following axioms.

Identity Axioms: There are elements 0 and 1 of F such that

$$x + 0 = x \qquad x \cdot 1 = x$$

for all $x \in F$.

Inverse Axioms: For all $x \in F$, there is an element $-x$ such that

$$x + (-x) = 0.$$

If x is a nonzero element of F, there is an element x^{-1} such that

$$x \cdot x^{-1} = 1.$$

Commutativity Axioms: For all x and y in F, we have

$$x + y = y + x \qquad x \cdot y = y \cdot x.$$

Associativity Axioms: For all x, y and z in F, we have

$$x + (y + z) = (x + y) + z \qquad x \cdot (y \cdot z) = (x \cdot y) \cdot z.$$

Distributivity Axiom: For all x, y and z in F, we have

$$x \cdot (y + z) = x \cdot y + x \cdot z.$$

Nontriviality Axiom: We have

$$0 \neq 1.$$