Vector, matrices, Image, kernel

Lecture 1

Quick reminder: vectors and matrices. (for first week, all scalars are real numbers; later, we generalize.)

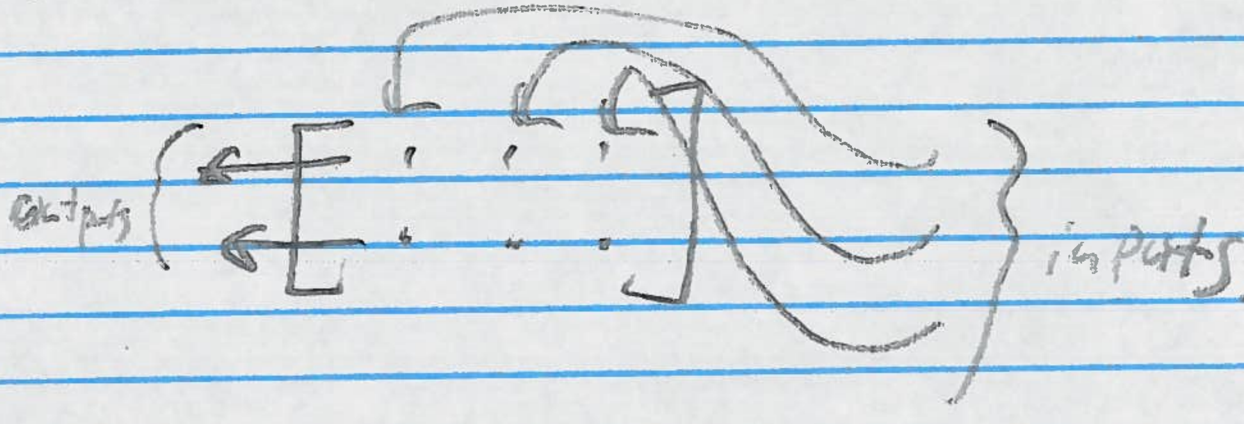
A vector is an n-tuple of real numbers $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$.

We add them component wise, and mult them by scalars.

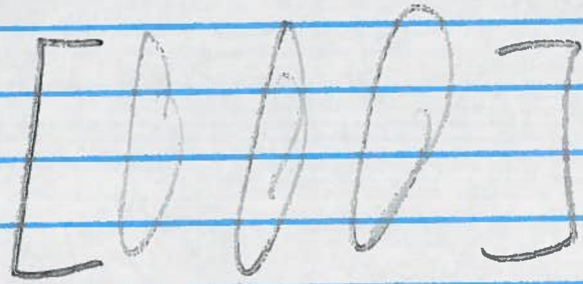
A ~~vector~~ matrix takes in one vector and makes another.

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} ax+by+cz \\ dx+ey+fz \end{bmatrix}$$

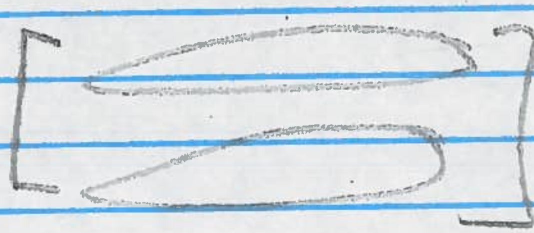
In general, think



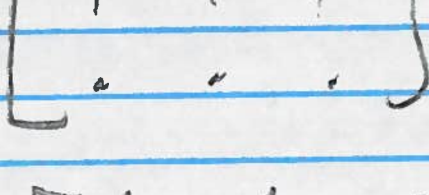
Terminology



rows



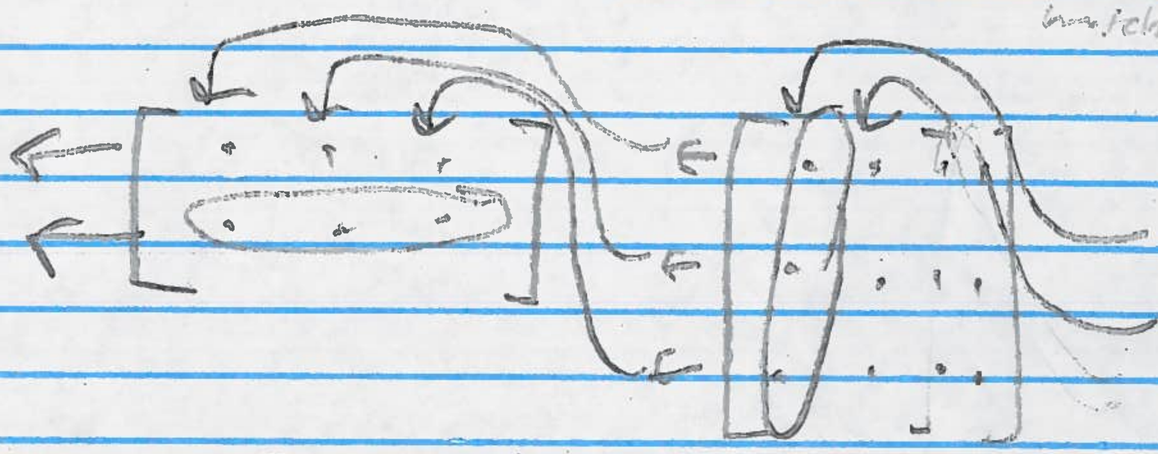
Columns



2×3

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix}$$

We multiply matrices whose sizes match



Or, in eqn $(AB)_{ik} = \sum_j A_{ij} B_{jk}$

We have

$$(AB)C = A(BC)$$

in particular

$$(AB)\vec{x} = A(B\vec{x}).$$

multiplication & composition

WORK SHEET Discuss at least Q1 and Q2

A reminder: Image and kernel.

Image (A) is set of all outputs of A.

i.e.

$$A = \begin{array}{c} \\ m \end{array} \left[\begin{array}{c} \overbrace{\qquad\qquad\qquad}^{n} \\ \\ \end{array} \right] \qquad A: \mathbb{R}^n \to \mathbb{R}^m$$

$$\text{Image}(A) = \{ \vec{y} \text{ in } \mathbb{R}^m : \exists \vec{x} \text{ in } \mathbb{R}^n \text{ with } \vec{y} = A\vec{x}$$

For example

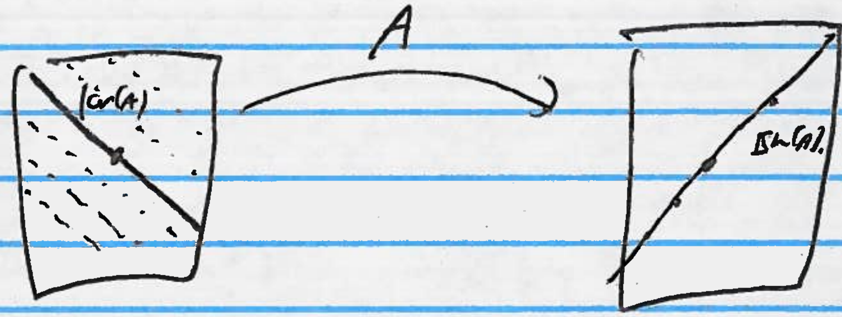
$$\text{Image} \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \text{ is } \begin{bmatrix} z \\ 3z \end{bmatrix}.$$

kernel (A) is the solutions to $A\vec{x} = 0$

$$\text{kernel}(A) = \{ \vec{x} \text{ in } \mathbb{R}^n : A\vec{x} = \vec{0} \}.$$

EG:

$$\text{kernel} \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \text{ is } \begin{bmatrix} -2w \\ w \end{bmatrix}.$$



~~Other reason~~

Image and kernel tell us how to solve eq'ns.

$A\vec{x}=\vec{b}$ has a solution iff $\vec{b}$ is in Image(A).

If $\vec{w}$ is one solution to $A\vec{w}=\vec{b}$, the other solutions are exactly $\vec{w}+\vec{k}$ for $\vec{k}$ in ker(A).

$\boxed{\text{Pf}}$ Let $\vec{x}=\vec{w}+\vec{k}$. Then

$$A\vec{x}=A(\vec{w}+\vec{k})=A\vec{w}+A\vec{k}=\vec{b}+A\vec{k}$$

So $A\vec{x}=\vec{b}$ iff $A\vec{k}=\vec{0}$.