Problem Set Three: Due Thursday, January 27 at 11:59 PM

See course website for homework policies.

Reading Read 2.4, 2.6 and A.4. Then fill out the poll at `https://forms.gle/oapfLCLdkbKqv3Lx6`

Textbook problems: We write $c.s.p$ as shorthand for "Chapter c, Section s, Problem p". Please solve the following problems:

$$2.2.2,\ 2.2.3,\ 2.2.8,\ 2.2.9,\ 2.3.1,\ 2.3.2,\ 2.3.3,\ 2.3.5$$

Problem 1. Let U, V and W be vector spaces (over some field F). Let f_1, f_2 be linear transformations $U \to V$ and let g_1 and g_2 be linear transformations $V \to W$.

(1) Show that $(g_1 + g_2)f_1 = g_1 f_1 + g_2 f_1$.
(2) Show that $g_1(f_1 + f_2) = g_1 f_1 + g_1 f_2$.

As a reminder, $f_1 + f_2$ is the linear transformation defined by $(f_1 + f_2)(\vec{u}) = f_1(\vec{u}) + f_2(\vec{u})$ and the other sums are defined likewise; the product $g_1 f_1$ is the linear transformation $U \to W$ defined by $(g_1 f_1)(\vec{u}) = g_1(f_1(\vec{u}))$.

Problem 2. Let A be a 2×4 real matrix with columns $\vec{a}_1$, $\vec{a}_2$, $\vec{a}_3$, $\vec{a}_4$ such that

- $\vec{a}_1$ is nonzero.
- $\vec{a}_2 = 3\vec{a}_1$.
- $\vec{a}_3$ is not in a multiple of $\vec{a}_1$.
- $\vec{a}_4 = 5\vec{a}_1 + 7\vec{a}_3$.

Find the row reduction of A and prove your answer to be correct.

Problem 3. Let (x_1, y_1), (x_2, y_2), $\ldots$, (x_6, y_6) be six points in the plane $\mathbb{R}^2$. By a quadratic polynomial, we mean a polynomial of the form $f(x, y) = f_{00} + f_{10}x + f_{01}y + f_{20}x^2 + f_{11}xy + f_{02}y^2$. Show that **exactly one** of the following conditions holds. Hint: What does this have to do with linear algebra?

- There is a nonzero quadratic polynomial $f(x, y)$ with $f(x_1, y_1) = \cdots = f(x_6, y_6) = 0$.
- For any 6 numbers z_1, z_2, $\ldots$, z_6, there is a quadratic polynomial $f(x, y)$ with $f(x_i, y_i) = z_i$.