Problem Set Five: Due Thursday, February 17 at 11:59 PM

See course website for homework policies.

Reading Read 3.5 and 3.7.

Textbook problems Please solve problems **3.2.6**, **3.2.8**, **3.4.8**, **3.5.2**, **3.5.8**

Problem 1. Let e_1, e_2, e_3 be the standard basis of $\mathbb{R}^3$. Let $f_1 = e_1$, $f_2 = e_2$ and $f_3 = e_1+e_2+e_3$. Express the dual basis vectors f_1^*, f_2^* and f_3^* as a linear combination of e_1^*, e_2^* and e_3^*. You should find that, even though $e_1 = f_1$ and $e_2 = f_2$, the dual vectors f_1^* and f_2^* are different from e_1^* and e_2^*.

Problem 2. Let $\vec{v}_1$,$\vec{v}_2$, $\dots$, $\vec{v}_n$ and $\vec{w}_1$, $\vec{w}_2$, $\dots$, $\vec{w}_n$ be two bases of a vector space V, and let v_1^*, v_2^*, $\dots$, v_n^* and w_1^*, w_2^*, $\dots$, w_n^* be the dual bases. Let the matrices A and B be defined by $\vec{w}_j = \sum_i A_{ij} \vec{v}_i$ and $w_j^* = \sum_i B_{ij} \vec{v}_i^*$. Show that $B = (A^T)^{-1}$.

Problem 3. Let C be the vector space of real polynomials of degree ≤ 3. For a real number r, let a_r be the function $f(x) \mapsto f(r)$ in C^*.

(1) Show that, if r_1, r_2, r_3, r_4 are four distinct real numbers, then a_{r_1}, a_{r_2}, a_{r_3}, a_{r_4} is a basis of C^*.
(2) Express the linear function $\int_0^3 f(x)dx$ as a linear combination of a_0, a_1, a_2 and a_3.