WORKSHEET 8: THE CHINESE REMAINDER THEOREM

> "*There are certain things whose number is unknown. If we count them by threes, we have two left over; by fives, we have three left over; and by sevens, two are left over. How many things are there?*" – Sunzi Suanjing (3rd century)

A lot of results today are quick citations to past worksheets! Have them ready!

Problem 8.1. Let R be a commutative ring and let A and B be ideals. Describe the "obvious" map $R \to R/A \times R/B$ and show that its kernel is $A \cap B$.

Problem 8.2. Show that, if R is a commutative ring and A and B are comaximal ideals, then $R/AB \cong R/A \times R/B$.

Problem 8.3. (**The Chinese Remainder Theorem**) Show that, if I_1, I_2, ..., I_k are a list of pairwise comaximal ideals, then

$$R/\left(I_1 I_2 \cdots I_k\right) \cong R/I_1 \times R/I_2 \times \cdots \times R/I_k.$$

Problem 8.4. Show that, if m_1, m_2, ..., m_k are a list of pairwise relatively prime integers, then

$$\mathbb{Z}/m_1 \cdots m_k\mathbb{Z} \cong \mathbb{Z}/m_1\mathbb{Z} \times \cdots \times \mathbb{Z}/m_k\mathbb{Z}.$$

Problem 8.5. Let k be a field and a_1, a_2, ..., a_r be distinct elements of k. Show that

$$k[t]/(t-a_1)(t-a_2)\cdots(t-a_r)k[t] \cong k \times \cdots \times k$$

where the right hand side has r factors.