

## THE CHINESE REMAINDER THEOREM

*“There are certain things whose number is unknown. If we count them by threes, we have two left over; by fives, we have three left over; and by sevens, two are left over. How many things are there?”* – Sunzi Suanjing (3rd century)

A lot of results today are quick citations to past worksheets! Have them ready!

(44) Let  $R$  be a commutative ring and let  $A$  and  $B$  be ideals. Describe the “obvious” map  $R \rightarrow R/A \times R/B$  and show that its kernel is  $A \cap B$ .

(45) Show that, if  $R$  is a commutative ring and  $A$  and  $B$  are comaximal ideals, then  $R/AB \cong R/A \times R/B$ .

(46) (**The Chinese Remainder Theorem**) Show that, if  $I_1, I_2, \dots, I_k$  are a list of pairwise comaximal ideals, then

$$R/(I_1 I_2 \cdots I_k) \cong R/I_1 \times R/I_2 \times \cdots \times R/I_k.$$

(47) Show that, if  $m_1, m_2, \dots, m_k$  are a list of pairwise relatively prime integers, then

$$\mathbb{Z}/m_1 \cdots m_k \mathbb{Z} \cong \mathbb{Z}/m_1 \mathbb{Z} \times \cdots \times \mathbb{Z}/m_k \mathbb{Z}.$$

(48) Let  $k$  be a field and  $a_1, a_2, \dots, a_r$  be distinct elements of  $k$ . Show that

$$k[t]/(t - a_1)(t - a_2) \cdots (t - a_r)k[t] \cong k \times \cdots \times k$$

where the right hand side has  $r$  factors.