Quiz 1: January 25

Problem 1. Let G be a group in which every element obeys $g^2 = 1$. Show that G is abelian.

Problem 2. Show that an abelian group A of order 100 cannot act faithfully on a set of size 13. By "faithfully", we mean that the map $A \to S_{13}$ has no kernel.

Quiz 2: February 1

Problem 3. Let n be a positive integer. Let G be the subgroup of S_n consisting of those maps $\mathbb{Z}/n\mathbb{Z} \to \mathbb{Z}/n\mathbb{Z}$ which are of the form $x \mapsto x + b$ or $x \mapsto -x + b$, for some $b \in \mathbb{Z}/n\mathbb{Z}$. How many conjugacy classes does G have? Your answer should depend on whether n is odd or even.

Problem 4. Let G be a group and let H be a subgroup with $\#(G/H) = n$. Show that there is a normal subgroup N with $N \subseteq H$ with $\#(G/N) \leq n!$.

Quiz 3: February 8

Problem 5. Does there exist a group G with normal subgroups N_1 and N_2 such that $N_1 \cong S_5$, $N_2 \cong S_7$, $G/N_1 \cong S_{42}$ and $G/N_2 \cong S_{41}$?

Problem 6. Classify all finite groups G whose only automorphism is the trivial automorphism.