For any commumative ring R, $\mathrm{SL}_2(R)$ is the group of 2×2 matrices with entries in R and determinant 1.

Problem 1 If a and b are relatively prime, show that

$$\mathrm{SL}_2(\mathbb{Z}/(ab)) \cong \mathrm{SL}_2(\mathbb{Z}/a) \times \mathrm{SL}_2(\mathbb{Z}/b).$$

Problem 2 Let p be prime. Show that the obvious map $\mathrm{SL}_2(\mathbb{Z}/p^k) \to \mathrm{SL}_2(\mathbb{Z}/p^{k-1})$ is surjective.

Problem 3 Describe the kernel of the map in the previous problem as an abstract group.

Problem 4 Show that $\mathrm{SL}_2(\mathbb{Z}/2) \cong S_3$.

Let $\mathbb{P}^1(\mathbb{F}_3)$ be the set of lines through the origin in the vector space $\mathbb{F}_3^2$ over the field with 3 elements. The action of $\mathrm{SL}_2(\mathbb{Z}/3)$ on $\mathbb{P}^1(\mathbb{F}_3)$ gives a map $\phi : \mathrm{SL}_2(\mathbb{Z}/3) \to S_4$.

Problem 5 Describe the kernel of ϕ as a well known group.

Problem 6 Describe the image of ϕ as a well known subgroup of S_4.

Problem 7 Show that $\mathrm{SL}_2(\mathbb{Z}/3)$ has order 24. Is $\mathrm{SL}_2(\mathbb{Z}/3)$ isomorphic to S_4?