Let S_3 act on $\mathbb{C}[r_1, r_2, r_3]$ by permuting the variables. Let

$$
\begin{aligned}
e_1 &= r_1 + r_2 + r_3 \\
e_2 &= r_1 r_2 + r_1 r_3 + r_2 r_3 \\
e_3 &= r_1 r_2 r_3
\end{aligned}
$$

Let $A_3 \subset S_3$ be the subgroup 123, 231, 312. Let $\omega = \frac{-1+\sqrt{-3}}{2}$, a primitive cube root of unity. Define $\chi : A_3 \to \mathbb{C}^*$ by $\chi(123) = 1$, $\chi(231) = \omega$, $\chi(312) = \omega^2$.

Question 1 Find a polynomial $F \in \mathbb{C}[r_1, r_2, r_3]$ such that $\sigma \cdot F = \chi(\sigma)f$ for $\sigma \in A_3$.

Question 2 How does S_3 act on F?

Question 3 How does S_3 act on F^3?