Define a **split** of $\{1, 2, 3, 4\}$ to be an unordered pair of two element sets whose union is $\{1, 2, 3, 4\}$.
So the splits are
$$\{\{1,2\}, \{3,4\}\} \qquad \{\{1,3\}, \{2,4\}\} \qquad \{\{1,4\}, \{2,3\}\}$$
The group S_4 acts on the set of splits, giving a map $\phi : S_4 \to S_3$.

Problem 1 Check that $\phi : S_4 \to S_3$ is surjective.

Problem 2 Let $V = \operatorname{Ker} \phi$. How large is V? List the elements of V.

Problem 3 Is V cyclic?