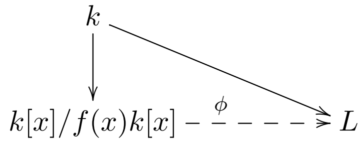
20. MAPS BETWEEN SPLITTING FIELDS

Problem 20.1. Let k be a field, let $f(x)$ be an irreducible polynomial in $k[x]$ and let L be an extension of k in which f has a root θ. Show that there is an injection $\phi : k[x]/f(x)k[x] \to L$ with $\phi(x) = \theta$ making the diagram

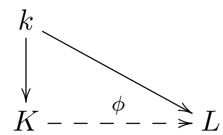


commute.

We recall the definition

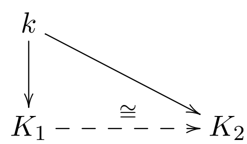
Definition: Let k be a field, let $f(x)$ be a polynomial in $k[x]$ and let K be an extension field of f. We will say that f ***splits in*** K if f factors as a product of linear polynomials in $K[x]$. We say that K is a ***splitting field of*** f if f splits as a product $c \prod (x - \theta_j)$ in $K[x]$ and the field K is generated by k and by the θ_j.

Problem 20.2. Let k be a field and let $f(x)$ be a polynomial in $k[x]$. Let K be a splitting field of f in which f splits as $\prod (x - \alpha_j)$. and let L be a field in which f splits as $\prod (x - \beta_j)$. Show that there is an injection $\phi : K \to L$ making the diagram



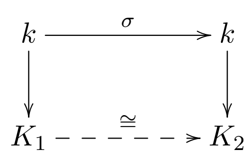
commute. Hint: Think about $k \subseteq k[\alpha_1] \subseteq k[\alpha_1, \alpha_2] \subseteq \cdots \subseteq k[\alpha_1, \alpha_2, \ldots, \alpha_n] = K$.

Problem 20.3. Let k be a field and let $f(x)$ be a polynomial in $k[x]$. Let K_1 and K_2 be two splitting fields of f. Show that there is an isomorphism $K_1 \cong K_2$ making the diagram



commute.

Problem 20.4. Let k be a field, let $f(x) = \sum f_j x^j$ be a polynomial in $k[x]$ and let σ be an automorphism of k. Let $\sigma(f)(x) := \sum \sigma(f_j) x^j$. Let K_1 be a splitting field of f and let K_2 be a splitting field of $\sigma(f)$. Show that there is an isomorphism $K_1 \cong K_2$ making the diagram



commute.