Problem Set Ten: Due Thursday, April 18.
Last problem set! Due on the last day of class!

See the course website for policy on collaboration.

Textbook problems:
18.2.H, 18.2 I: These justify the assertion from class that we can often base change to $\overline{k}$ before thinking about projective embeddings. (Problem 19.2.E is another problem with this theme but I'm not assigning it because it relies on too many earlier problems that I didn't assign.)
19.2.A: We'll use this in Problem 1 below.
19.8.C This exercise has you think about curves of genus five, in the same way that I plan to cover curves of genus 4.

Problem 1. Here are three problems where we can describe the canonical sheaf of a curve directly, without mentioning differentials, by using Problem 19.2.A.

(1) Let X be a smooth projective irreducible curve of genus g and let $\pi : X \to \mathbb{P}^1$ be a degree 2 map. Show that $\pi^*(\mathcal{O}(g-1)) \cong \omega_X$.
(2) Let Y be a smooth projective irreducible degree d curve in $\mathbb{P}^2$; we write ι for the inclusion $Y \hookrightarrow \mathbb{P}^2$. Show that $\iota^*\mathcal{O}(d-3) \cong \omega_C$. (Hint: As we so often do, start with $0 \to \mathcal{O}(-d) \to \mathcal{O} \to \iota^*\mathcal{O} \to 0$.)
(3) Let f and g be homogeneous polynomials of degrees a and b in $k[x_0, x_1, x_2, x_3]$, and let $Z = V_+(f, g)$; suppose that Z is smooth and irreducible. In , Problem 3, Problem Set 9, you hopefully computed that Z has genus $(a^2b+ab^2-4ab+2)/2$. Show that $\iota^*\mathcal{O}(a+b-4) \cong \omega_Z$.