Obvious assertions for the week of January 7-14

(1) Let X be a topological space and let $\mathcal{E}$ be a sheaf on X. Let U be a open subset of X and let f and $g \in \mathcal{E}(U)$. Show that, if f and g represent the same element of the stalk $\mathcal{E}_x$ for all $x \in U$, then $f = g$. **Used on Jan 21 quiz**

(2) Let $X = \{(x, y) \in \mathbb{R}^2 : 1 < x^2 + y^2 < 2\}$. Let $\mathcal{E}$ be the sheaf of continuous $\mathbb{C}$ valued functions on X. Show that $U \mapsto \exp(\mathcal{E}(U))$ is not a sheaf of $\mathbb{C}$-valued functions on X.

(3) Let X be a topological space and let $\mathcal{E}$ be a presheaf on X. Let F be the set of pairs (U, f) where U is an open set containing x and $f \in \mathcal{E}(U)$. Define a relation $\sim$ on F by $(U_1, f_1) \sim (U_2, f_2)$ if there exists some $(U_3, f_3) \in F$ with $U_3 \subset U_1 \cap U_2$ and $\rho^{U_1}_{U_3}(f_1) = f_3 = \rho^{U_2}_{U_3}(f_2)$. Show that $\sim$ is an equivalence relation. **Used on Jan 21 quiz**

(4) Let X be a topological space and $\mathcal{E}$ a sheaf on X. Let x be a point of X and let $\mathcal{V}$ be a basis of neighborhoods of x. (Meaning that the elements of $\mathcal{V}$ are open sets containing x and, for any open set $U \ni x$, there is some $V \in \mathcal{V}$ with $x \ni V \subseteq U$.) Show that $\varinjlim_{v \in \mathcal{V}} \mathcal{E}(V)$ is isomorphic to the stalk $\mathcal{E}_x$.

Obvious assertions for the week of January 14-21

(5) Let X be a topological space, and let E be a set. Let $\mathcal{E}$ a presheaf of E-valued functions on X. For every open subset U of X, define $\mathcal{E}^+(U)$ to be the set of function $f : U \to E$ so that, for every $x \in U$, there is an open set V with $x \in V \subset U$ so that $f|_V \in \mathcal{E}(V)$.

Show that $\mathcal{E}^+$ is a sheaf. **Used on Jan 21 quiz**

Obvious assertions for the week of January 21-28

(6) Let A be a commutative ring. Recall that the Zariski topology on $\operatorname{Spec} A$ is defined as follows: For any $S \subseteq A$, we define $V(S) = \{\mathfrak{p} \in \operatorname{Spec} A : S \subset \mathfrak{p}\}$. The $V(S)$ are the closed sets of the Zariski topology. Show that this defines a topology on $\operatorname{Spec} A$.

(7) Let A and B be commutative rings and let $\phi : A \to B$ be a map of commutative rings. Let $\mathfrak{p}$ be a prime ideal of B. Show that $\phi^{-1}(\mathfrak{p})$ is a prime ideal of A.

(8) Let A and B be commutative rings and let $\phi : A \to B$ be a map of commutative rings. Show that the map $\mathfrak{p} \to \phi^{-1}(\mathfrak{p})$ is a continuous map $\operatorname{Spec} B \to \operatorname{Spec} A$. (You may assume that $\phi^{-1}(\mathfrak{p})$ is prime.)

(9) Let A be a commutative ring and let f be an element of A. Show that the natural map $\operatorname{Spec} f^{-1}A \to \operatorname{Spec} A$ is an inclusion with image $D(f) = \{\mathfrak{p} : f \not\in p\}$.

(10) Let A be a commutative ring. Recall that the distinguished open $D(f)$ is $\{\mathfrak{p} \in \operatorname{Spec} A : f \not\in \mathfrak{p}\}$. Show that $D(f) \cap D(g) = D(fg)$.