Let X be a topological space and let E be a set. Let U_i be a basis of open sets on X, meaning that, for every open set V in X and every $x \in V$, there is a U_i with $x \in U_i \subset V$. For each U_i, let $\mathcal{E}(U_i)$ be a collection of E-valued functions on U_i.

Suppose that

(1) If $f \in \mathcal{E}(U_i)$ and $U_j \subset U_i$, then $f|_{U_j} \in \mathcal{E}(U_j)$.

(2) If $U_i = \bigcup_{j \in J} U_j$ and f is a function $U_i \to E$ such that $f|_{U_j} \in \mathcal{E}(U_j)$ for each $j \in J$, then $f \in \mathcal{E}(U_i)$.

Show that there is a unique sheaf $\mathcal{F}$ of E valued functions on X such that $\mathcal{F}(U_i) = \mathcal{E}(U_i)$ for each U_i.