

This problem tries to make explicit the computations that are implicit in Hartshorne's II.6.9. Let  $k$  be a field of characteristic not equal to 2 or 3.

Let  $X$  be the curve  $y^2z = x^3$  in  $\mathbb{P}_k^2$  and let  $Y$  be the curve  $y^2z = x^3 - x^2z$ .

(a) Find the smooth points of  $X$  and  $Y$  respectively. Give an isomorphism between the smooth locus of  $X$  and  $\mathbb{A}^1 := \text{Spec } k[t]$ . Give an isomorphism between the smooth locus of  $Y$  and  $\mathbb{G}_m := \text{Spec } k[u, u^{-1}]$ .

(b) In terms of the parametrizations in (a), give a formula for when three distinct smooth points of  $X$  and  $Y$  (respectively) are collinear. You should get a nice answer!