MATH 665 PROBLEM SET 2: DUE SEPT 24

See the course website for homework policy.

Problem 1 Compute $s_{5,3}(x, y, z)$ (explicitly write out a sum of monomials). This can look quite pretty if you organize your answer well.

Problem 2

(a) Show that the coefficient of $x^p y^q$ in $s_{ab}(x, y)$ is 0 or 1, and describe when each case occurs.

(b) Give a formula for the coefficient of $x^p y^q z^r$ in $s_{abc}(x, y, z)$. Your answer should be a piecewise linear function of (p, q, r).

Problem 3 This problem verifies that the Schur functions are a basis for Λ.

(a) Show that, if the monomial $x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_r^{\alpha_r}$, occurs in $s_\lambda(x_1, \dots, x_r)$, then $\alpha \preceq \lambda$. (Recall that $\alpha \preceq \lambda$ means $\alpha_1 \leq \lambda_1$, $\alpha_1 + \alpha_2 \leq \lambda_1 + \lambda_2$, ..., $\alpha_1 + \alpha_2 + \cdots + \alpha_r \leq \lambda_1 + \lambda_2 + \cdots + \lambda_r$.)

(b) Show that the coefficient of $x_1^{\lambda_1} x_2^{\lambda_2} \cdots x_r^{\lambda_r}$ in s_λ is 1.

Problem 4

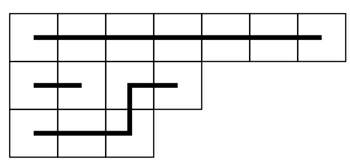
Let λ be a partition and let $a_1 + a_2 + \cdots + a_i + b + c_1 + c_2 + \cdots + c_j = |\lambda|$. Let f_k be the coefficient of

$$x_1^{a_1} x_2^{a_2} \cdots x_i^{a_i} x_{i+1}^{k} x_{i+2}^{b-k} x_{i+2}^{c_1} \cdots x_{i+j+2}^{c_j}$$

in s_λ. Show that

$$f_0 \leq f_1 \leq f_2 \leq \cdots \leq f_{\lfloor b/2 \rfloor} = f_{\lceil b/2 \rceil} \geq \cdots \geq f_{b-1} \geq f_b$$

Problem 5 Let λ be a partition. A ***rim hook*** is a path through λ, starting at the left hand edge, and moving left and upwards one square at a time. A ***rim hook tableau*** is a decomposition of λ as a disjoint union of rim hooks. The ***weight*** of a rim hook tableaux is the partition formed by sorting the rim hooks into order. For a rim hook tableau T, let $\ell(T)$ be the number of vertical steps in T.



A rim hook tableau of shape $(7, 4, 3)$ and weight $(7, 5, 2)$, with $\ell(T) = 1$

(a) Show $K^{-1}_{\alpha\beta}$ is the sum over rim hook tableaux T of shape α and weight β of $(-1)^{\ell(T)}$.

(b) Prove or disprove: For fixed partition α and weight β, all rim hooks of shape α and weight β contribute the same value of $(-1)^{\ell(T)}$ (so there is no cancellation in the above sum).