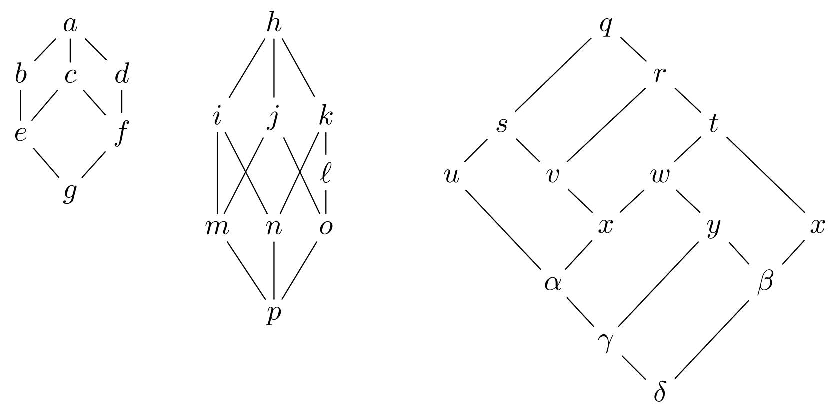
# Problem Set 8: Due Friday, November 8

See the course website for homework policy.

1. Please TeX up a solution to your assigned problem and send it to Professor Speyer.

2. The following pictures show the Hasse diagrams of some small lattices. In each case, determine the slide equivalence classes on Covers($L$), and the forcing relations between them:




3. Let $W$ be a finite Coxeter group with simple generators $s_1, s_2, \ldots, s_n$. For $w \in W$, let $D(w) = \{k : s_k \text{ is a left descent of } w\}$. Let $2^{[n]}$ be the lattice of subsets of $\{1, 2, \ldots, n\}$, with the usual order. Show that $w \mapsto D(w)$ is a lattice homomorphism $W \to 2^{[n]}$.

4. (a) Show that an element of $S_n$ is join irreducible if and only if it has exactly one right descent.
   (b) Show that there are $2^n - n - 1$ join irreducible elements in $S_n$.

5. This question shows that trying to define something like weak order for hyperplane arrangements not coming from root systems can lead to problems. Let $R$ be the following subset of $\mathbb{R}^4$:

$$R = \left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\\1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\\1\end{bmatrix} \right\}.$$

   Define a subset $I$ of $R$ to be *convex* if there is a $\theta \in \mathbb{R}^4$ such that $I = \{\beta \in R : \langle \theta, \beta \rangle > 0\}$. We treat the set of closed sets as a poset under containment. In this problem, we will show that this poset is not a lattice.

   (a) Show that the following sets are closed:

$$\left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix} \right\}, \left\{ \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\\1\end{bmatrix} \right\}, R - \left\{ \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\1\end{bmatrix} \right\} \text{ and } R - \left\{ \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\\1\end{bmatrix} \right\}.$$

   (b) Show that

$$\left\{ \begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\\0\end{bmatrix} \right\} \vee \left\{ \begin{bmatrix}0\\0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\\1\end{bmatrix} \right\}$$

   does not exist in the poset of convex sets.