# Problem Set 9: Due Friday, November 15

See the course website for homework policy.

1. Please read the course notes for November 1-13 and suggest a question or something that could be improved.

2. This problem checks some lemmas that we omitted from our discussion of doubling. Let $L$ be a lattice and $I = [a, b]$ an interval of $L$. Define the doubling $L[I]$ to have the ground set $(L \setminus I) \sqcup (I \times \{0, 1\})$. Let $\pi : L[I] \to L$ be the obvious map. Define a partial order on $L[I]$ by setting $x \leq y$ if

   - $\pi(x) \leq \pi(y)$ and
   - in the case that $x = (\pi(x), i)$ and $y = (\pi(y), j)$ are both in $I$, we additionally require that $i \leq j$.

   (a) Check that $L[I]$ is a lattice.

   (b) Check that $\pi$ is a map of lattices.

   (c) Let $L$ be finite. Check that the join irreducible elements of $L[I]$ are

   - Join irreducible elements $j$ of $L$ which are not in $I$
   - Pairs $(j, 0)$, for $j$ a join irreducible element of $L$ which is in $I$
   - The element $(a, 1)$.

3. This question investigates two quotients of $S_4$ weak order which are useful as counter-examples.

   (a) Consider the covers $(2143, 2413)$ and $(1423, 4123)$. Compute the slide equivalence class of each, and show that neither forces any covers outside of its slide equivalence class.
   Let $L_1$ and $L_2$ be the quotients of $S_4$ where we contract the slide equivalence classes of $(2143, 2413)$ and $(1423, 4123)$ respectively.

   (b) Draw the coarsening of the $S_4$ hyperplane arrangement corresponding to $L_1$ and $L_2$. (You may want to download the stereographic projection of $A_3$ from the course website.) Note that $L_1$ has a non-simplical cone.

   (c) Show that none of the right inverses of $\pi : S_4 \to L_2$ is a map of lattices.